# Refinement of the Crystal Structure of Beryllium Sulphate Tetrahydrate 

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#### Abstract

Refinement of the crystal structure of $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}$ by X-ray methods has confirmed the results of an earlier analysis. The unit cell is tetragonal with $a=7.990 \AA, c=10.688 \AA$, space-group $I 4 c 2$. The lattice is built up from $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2+}$ and $\mathrm{SO}_{4}^{2-}$ ions. The bond distances are $\mathrm{S}-\mathrm{O}=1 \cdot 465$ and $\mathrm{Be}-\mathrm{OH}_{2}=$ $1.610 \AA$, with estimated standard deviations $0.004 \AA$. When corrected for differential thermal vibration, these distances become 1.482 and $1.622 \AA$, respectively. The tetrahedral $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}$ and $\mathrm{SO}_{4}$ groups both possess slight angular distortions from $T_{i}$ symmetry. The hydrogen atoms could not be located with precision but the probable geometry of the water molecule can be deduced from the hydrogenbonding. The crystals exhibit strong diffuse X-ray scattering.


## Introduction

The crystal structure of $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ was determined by Beevers \& Lipson (1932). The arrangement of the tetrahedral $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2+}$ and $\mathrm{SO}_{4}^{2+}$ ions is based on a slightly elongated cesium chloride lattice. Each $\mathrm{SO}_{4}^{2+}$ ion is surrounded by eight $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2+}$ ions, and each O (sulphate) atom is hydrogen-bonded to $\mathrm{H}_{2} \mathrm{O}$ molecules belonging to two of these.

The purpose of the present refinement was the accurate determination of the dimensions of the sulphate ion. $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}$ was selected because, as in $\mathrm{Li}_{2} \mathrm{SO}_{4}$. $\mathrm{H}_{2} \mathrm{O}$ (Larson, 1965), the X-ray scattering power is dominated by the contribution of the sulphate ion. Unlike $\mathrm{Li}_{2} \mathrm{SO}_{4} . \mathrm{H}_{2} \mathrm{O}$, however, the beryllium salt has a crystal structure in which the sulphate ions are held by forces which cause a minimal perturbation of their symmetry and dimensions. In this sense the present work complements that of Larson (1965).

## Procedure

## Experimental

The salt ('AnalaR' grade), crystallized from ca .10 N sulphuric acid solution, was sealed in Lindemann glass capillary tubes for X-ray examination (because of its toxicity). $\mathrm{Cu} K \alpha$ radiation was used. The unit-cell dimensions were fitted by Cohen's least-squares method (Buerger, 1942) to the $\sin \theta$ values of 51 high-angle reflexions, measured on powder photographs recorded in the Straumanis mode. The probable errors, after allowing for systematic absorption and eccentricity effects, have been shown to be three times the statistical estimated standard deviations.

$$
\begin{array}{ll}
\mathrm{BeH}_{8} \mathrm{O}_{8} \mathrm{~S} \\
\text { Tetragonal: } & a=7 \cdot 990 \pm 1, c=10 \cdot 688 \pm 6 \AA, \quad \text { F.W. }=177 \cdot 14 \\
& V=682 \cdot 3 \AA^{3}, z=4, D_{x}=1 \cdot 733 \text { g.cm }{ }^{-3},
\end{array}
$$

[^0]\[

$$
\begin{gathered}
D_{m}=1.725 \text { (Töpsöe, 1872), } 1.712 \mathrm{g.cm}^{-3} \\
\text { (Krüss \& Moraht, 1890) } \\
\mu=44.5 \mathrm{~cm}^{-1}, \begin{array}{l}
\lambda\left(\mathrm{Cu} \mathrm{~K} \alpha_{1}\right)=1.5405 \AA, \\
\lambda\left(\mathrm{Cu} \mathrm{~K} \mathrm{\alpha}_{2}\right)=1.5443 \AA .
\end{array} .
\end{gathered}
$$
\]

Space-group $I \overline{4} c 2$ (Beevers \& Lipson, 1932) confirmed by the systematic absences ( $h k l, \quad h+k+l=2 n+1$; $h 0 l, l($ and $h)=2 n+1)$, and structure analysis.

Intensity data for layers $h=0$ through $h=6$ and $l=0$ through $l=3$ were recorded photographically and estimated visually. Of the 229 unique reflexions accessible with $\mathrm{Cu} K \alpha$ radiation, one was not recorded and four were unobservably weak. Absorption corrections were applied. The high lattice symmetry of the crystal and the extensive photographic recording resulted in multiple intensity estimates for most reflexions. The standard deviations $\sigma\left(F_{o}\right)$ of the structure factors $\left|F_{0}\right|$ were derived by applying standard statistical formulae (Whittaker \& Robinson, 1944; Davies \& Pearson, 1934) to three sets of multiple data: (i) the observations of a single reflexion on the films of a film-pack, (ii) the sym-metry-related reflexions recorded on each layer, and (iii) symmetry-related reflexions occuring on different layers or recorded about different axes. Observations of types (i) and (ii) were combined to yield an estimated variance $\sigma_{1}$, considered to represent mainly random errors, whereas $\sigma_{2}$ from the less numerous data in class (iii) was regarded as representing both systematic and random errors. The value of $\sigma\left(F_{o}\right)$ was assigned as the larger of $\sigma_{1}$ and $\sigma_{2}$, subjectively adjusted in a few cases where either of these values deviated disproportionately from the general trends.

## Refinement of the structure

The sulphur and beryllium atoms lie in special positions (b) and (c) with point symmetry $\overline{4}$ - the $S$ atoms at $(0,0,0),\left(0,0, \frac{1}{2}\right),\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right),\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and the Beatoms at $\left(0, \frac{1}{2}, \frac{1}{4}\right)$, $\left(0, \frac{1}{2}, \frac{3}{4}\right),\left(\frac{1}{2}, 0, \frac{1}{4}\right),\left(\frac{1}{2}, 0, \frac{3}{4}\right)$. The tetrahedra are formed by sets of oxygen atoms in 16 -fold general positions.
$F_{o}$ and ( $F_{o}-F_{c}$ ) Fourier syntheses confirmed the approximate atomic positions obtained by Beevers \&
Table 1. Positional and vibrational parameters
The standard deviations (in parentheses) refer to the two least significant digits in the preceding quantities. $B_{\text {iso }}$ is the isotropic vibrational parameter for the hydrogen atoms. The anisotropic atomic temperature factors are.
$\left.\left.* a^{*} U_{31}\right)\right]$.
$U_{12}\left(\AA^{2}\right)$
0
0
$0.0058(16)$
$-0.0085(16)$
-
$U_{33}\left(\AA^{2}\right)$
$U_{22}\left(\AA^{2}\right)$
$0.0148(13)$
$0.0200(56)$
$0.0219(19)$
$0.0205(19)$
-
$U_{11}\left(\AA^{2}\right)$
$0.0148(13)$
$0.0200(56)$
$0.0281(21)$
$0.0340(23)$
-
$\left(\AA^{2}\right)$
$\left(\AA^{2}\right)$
-
-
-
$2 \cdot 5$
$2 \cdot 5$
$\left.\sigma_{z}\right)$

$\left(\sigma_{z}\right)$
N
0.077
0.17182
0.130
$0 \cdot 130$
$0 \cdot 125$
$\sigma_{\left(\sigma_{\nu}\right)}$
0.465
0.281





Lipson (1932), and indicated markedly anisotropic thermal motions for the sulphur and oxygen atoms. The initial refinements of the positional parameters of the oxygen and the anisotropic thermal parameters of all the atoms were made by Fourier methods. The residual $R$ was reduced to 0.075 . Diffuse regions of positive electron density appeared in the expected vicinities of the hydrogen atoms, which were therefore assigned tentative positions and isotropic thermal parmeters $B=2.5 \AA^{2}$.

In the full-matrix least-squares refinement, the reflexions were given individual weights $w=1 / \sigma^{2}\left(F_{o}\right)$, and the function minimized was $\Sigma w\left|\left|\mathrm{~F}_{o}\right|-\left|F_{c}\right|^{2}\right.$. The atomic scattering factors used were those of Dawson (1960) for sulphur, Berghuis, Haanappel, Potters, Loopstra, MacGillavry, \& Veenendaal (1955) for oxygen, Ibers (1957) for beryllium and McWeeny (1951) for hydrogen. Dispersion corrections were applied to the sulphur scattering curve (International Tables for X-ray Crystallography, 1962). All except the hydrogen atom parameters were refined. Appropriate constraints were imposed on the thermal parameters of atoms in special positions. The refinement converged after four cycles ( $R=0.072$ ).

Re-inspection of the data showed that there was a systematic trend in the remaining discrepancies due to extinction, for which corrections were applied according to the method of Pinnock, Taylor \& Lipson (1956). An analysis of the weighting scheme at this stage showed that the mean value of $w^{\ddagger} \Delta$ was $2 \cdot 0$, and independent of the magnitude of $\left|F_{o}\right|$. Individual $w^{\frac{1}{\Delta}} \Delta$ values, however, deviated strongly from the mean, the standard deviation of the mean value 2.0 for the population being $1 \cdot 8$. This indicated that the $\sigma\left(F_{o}\right)$ had been assessed with insufficient precison to validate their use in assigning weights to the $F_{0}$. The least-squares refinement was therefore completed with a weighting derived from the variation of root mean squares $\left|\left|F_{0}\right|-\left|F_{c}\right|\right|$ in ranges of $\left|F_{o}\right|$. Three cycles were necessary, converging with $\Sigma w \Delta^{2} /(m-n)=1.046$ and residuals $R$ (unweighted $=$ 0.048 and $R^{\prime}$ (weighted) $=0.069$. The most significant features in a final ( $F_{o}-F_{c}$ ) Fourier synthesis, the hydrogen contributions having been omitted from the $F_{c}$ 's, were broad peaks in the regions of the hydrogen atoms. This observation and the failure of further refinement to reduce the residual $R$ led to the conclusion that the X-ray diffraction data could not yield the hydrogen positions with errors smaller than $0 \cdot 1 \AA$.

The atomic parameters and their calculated standard deviations are given in Table 1, and the final structore factors are listed in Table 2. We use the symbols O and $\mathrm{O}_{w}$ or $\mathrm{O}(1)$ and $\mathrm{O}(2)$ for the O (sulphate) and O (water) atoms, respectively. The two independent H (water) atoms are $\mathrm{H}(1)$ and $\mathrm{H}(2)$.

## Description of the structure

It follows from the special positions occupied by the S and Be atoms that the tetrahedral $\mathrm{SO}_{4}^{2-}$ and

Table 2. Calculated and observed structure factors $F(000)=172$. Unobservably weak reflexions are indicated by $X$.

| $\frac{y}{8}$ | $\stackrel{\underline{k}}{x_{\text {cale }}}$ | Pbs |  | $\underline{\underline{1}} \underset{y_{\text {cale }}}{\underline{k}}$ | $F_{\text {oba }}$ |  | $\underset{r_{\text {cale }}}{\mathbf{k}}$ | $F_{\text {obe }}$ | $\stackrel{b}{f}$ | $\underset{r_{\text {eale }}}{\underline{E}}$ | ${ }^{\text {abob }}$ |  | $\frac{n}{f} \stackrel{s}{r_{\text {cale }}}$ | $F_{o b 0}$ | $\frac{\mathrm{n}}{\underline{l}}$ | $\stackrel{\underline{k}}{\mathrm{~F}_{\text {eale }}}$ | ${ }^{\text {obe }}$ | $\frac{\mathrm{h}}{\underline{l}}$ | $\underset{\mathbf{r}_{\text {enlo }}}{\mathbf{t}}$ | ${ }_{\text {Obs }}$ | $\underline{\square}$ | $\stackrel{\underline{E}}{\mathbf{P}_{\text {oule }}}$ | $P_{\text {obo }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 |  | 1 | 11 |  | 1 | 6 |  | 2 | 3 |  | 2 | 9 |  | 3 | 7 |  | 4 | 7 |  | 5 | 8 |  |
| 2 | 39.89 | 45.86 |  |  |  | 1 | 17.69 | ${ }^{10} \cdot 21$ |  |  |  |  |  |  | 3 |  |  | 4 | 7 |  |  |  |  |
| 6 | 24.30 29.46 | 24.14 31.39 |  | 2126.51 | 227.94 25 35 | 5 | 21.08 6.49 | 20.25 6.11 | 3 | 64.24 | 50.27 | 3 | 16.57 6.03 | 16.33 5.00 | ? | 28.62 29.18 | 29.22 | $\frac{1}{3}$ | 7.24 5.12 | 7.32 6.18 | 3 | 6.58 8.65 | 7.44 |
| ${ }^{8}$ | 11.77 | 10.69 |  | 33.76 19.39 | 35.25 16.96 | 7 | 226 | ${ }_{2}^{6.44} \mathbf{7}$ | 5 | ${ }^{22} 4.12$ | 20.02 3.01 | 5 | 12.33 | 12.35 | 4 | ( 20.14 | 18.82 | 5 | 7.21 <br> 3.76 | 6.18 4.95 9.45 | 5 | 10.54 | 21.02 |
| 12 | $\underset{\substack{16.28 \\ 35.33}}{ }$ | 114.40 38.06 | 8 | 18.17 | 18.67 |  | 7.72 0.87 |  | 9 | 24.19 | 23.65 |  |  |  | 6 | 20.42 | 18.76 |  | 8.85 | 9.45 |  |  |  |
|  |  | 38.06 | 12 | $\begin{gathered} 30.78 \\ 6.03 \end{gathered}$ | $\begin{aligned} & 30.30 \\ & 11.30 \end{aligned}$ |  |  |  | 11 | 10.26 | 10.32 | 2 | 10 |  | 8 |  |  |  |  |  | 5 | 9 |  |
| 0 | 2 |  |  |  |  | 1 | 7 |  | 2 | 4 |  | $\stackrel{1}{2}$ | $\begin{aligned} & 15.06 \\ & 11: 02 \end{aligned}$ | 15.20 13.58 | 3 | 8 |  | 4 | 8 |  | $\bigcirc$ | 10.54 | 12.56 |
| 0 | 97.46 | 92.51 | 1 | 2 |  | $\stackrel{0}{ }$ | 13.93 | 23.19 |  |  |  |  |  |  | 1 | 7.53 | 7.40 | $\bigcirc$ | 12.99 | 13.94 |  |  |  |
| 2 | 29.34 | 644.71 |  |  |  |  | 33.80 30.21 | 34.38 31.23 | 1 | 28.43 | ${ }_{28} 8.19$ | 3 | 3 |  | 3 | 7.72 | 7.28 | 2 | 23.18 | 21.56 22.79 | 6 | 6 |  |
| 6 | 16.85 | 18.18 | 1 | 34.75 75.79 | 37.05 75.74 | 6 | 20.51 | 11.23 19.19 | 4 | 40.74 27.86 | 42.46 26.75 |  |  |  | 5 | 13.94 | 12.58 | 6 | 6.49 | 6.44 | 0 | 19.39 | 10.73 |
| 8 | 27.58 | 27.64 | 5 | 34.24 | 34.86 | 8 | 19.10 | 19.15 | 8 | 28.80 | 28.02 | $\bigcirc$ | 10.73 | 11.34 |  |  |  |  |  |  | 2 | 17.51 | 17.84 |
| 12 | 19.67 20.99 | 17.55 22.28 |  | 12.71 | 11.56 | 10 | 12.05 | 12.78 | 10 | 15.82 | ${ }_{15}^{15.66}$ | $\stackrel{2}{4}$ | 55.15 29.46 | 55.43 29.70 | 3 | 9 |  | 4 | 9 |  | 6 | 10.59 10.54 | 18.93 9.92 |
|  |  |  | 11 | 10.17 | 16.38 0.34 0 |  |  |  |  |  |  | 6 | 22.97 | 20.89 | 3 | 9 |  |  |  |  |  |  |  |
| 0 | 4 |  | 13 | 1.32 | 1.18 I | 1 | $\bigcirc$ |  | 2 |  |  | 10 | 18.73 | 23.67 18.24 | $\bigcirc$ | 19.67 | 20.84 | $\frac{1}{3}$ | 2.07 5.17 | 2.15 | 6 | 7 |  |
|  |  |  |  |  |  |  | 9.42 | 8.25 | 2 | 5 |  | 12 | 9.04 | 7.94 | 4 | 13.09 | 13.27 |  |  |  |  |  |  |
| ${ }_{2}$ | 66.94 36.76 | 64.01 37.40 | 1 | 3 |  | 5 | 11.67 10.45 | 10.49 10.16 1.98 | $\frac{1}{3}$ | 22.10 | 20.65 |  |  |  |  |  |  | 5 | 5 |  | 3 | ${ }^{11} 7.86$ | 21.39 |
| 4 | 64.57 | 65.57 | 0 |  |  | 7 | 11:86 | 11.98 | 5 | 22.78 | 21.22 | 3 | 4 |  | 4 | 4 |  | 5 | 27 |  | 5 | 2.45 | 1.36 |
| 6 | 17.41 | 15.44 | 2 | 14.31 |  |  |  |  | 7 | 14.21 | 12.74 |  |  |  |  |  |  | 0 | 27.02 | 27.17 |  |  |  |
| 8 | 54.50 | 55.73 | 4 | 34.65 | 36.29 |  |  |  | 9 | 4.33 | 12.84 | 1 | 16.00 | 15.45 | 0 | 90.62 | 90.53 | 2 | 40.61 | +0.48 | 6 |  |  |
| 12 | 17.23 | 16.74 | ${ }_{6}^{6}$ | 58.57 | 63.47 |  |  |  | 11 | 3.20 | 3.36 | 5 | 12.14 | 11.15 | 2 | 17.33 | 18.77 | 6 | 4.62 | 4.45 | 6 |  |  |
|  |  |  |  | 23.92 | 23.33 |  | 9 |  |  |  |  | 7 | 17.51 | 11.13 | ${ }_{6}$ | 20.43 | ${ }^{1} 8.12$ | $\stackrel{\circ}{8}$ | 10.26 | 10.40 | $\bigcirc$ | 11.39 | 12.58 |
| 0 | 6 |  | 12 | 14.12 | 13.73 | $\bigcirc$ | 13.46 | ${ }^{14 .} 38$ | 2 | 6 |  | $2{ }^{9}$ | 4.05 | 3.3 | [888 | 4.89 10.83 | 4.70 9.77 | 10 | 10.45 | 23.08 | 2 |  |  |
|  |  |  |  |  |  | 4 | 16:57 | 15.78 | 0 |  | 34.32 |  |  |  |  |  |  |  |  |  |  |  |  |
| $\stackrel{1}{2}$ | ${ }^{6.21}$ | 5.97 | 1 | 4 |  | 6 | 9.42 | 9.45 | 2 | 22.69 | 21.26 | 3 | 5 |  |  |  |  | 5 | 6 |  | 7 | 7 |  |
| 4 | 43.91 | 35.02 45.02 | 1 |  |  |  |  |  | 6 | 31.44 10.96 | 31.24 19.93 |  |  |  | 4 | 5 |  |  | 9.04 | 9.74 | 0 | 7.81 | 7.60 |
| 8 | $\xrightarrow{25.15}$ | 27.42 24.58 | 3 | 22.49 | 20.12 | 1 | 10 |  | 8 | 17.32 | 17.71 | ${ }_{2}$ | 10.83 51.30 | 17.42 51.40 | $\frac{1}{3}$ | 9.22 | 8.49 | 5 | 9.22 | 8.52 9.51 |  |  |  |
| 10 | 19.10 | 18.68 | 7 | 19.01 | 17.45 |  |  |  | 10 | 11.49 | 10.32 | 4 | 34:74 |  |  | 11.95 |  | 7 | 3.58 | ${ }^{3} 58$ |  |  |  |
|  |  |  |  | 18.07 | 17.69 | 3 | 8.19 | 8.19 2 |  |  |  | 6 | 9:00 | 6.54 | 7 | 9.22 | 13.69 | 9 | 4.05 | 4.86 |  |  |  |
| 0 | 8 |  | 11 | $1: 89$ | 1.501 |  |  |  |  |  |  | $\stackrel{8}{8}^{8}$ | 17.41 | 27.65 | 9 | 2.45 | 2.25 I |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 2 | 7 |  |  |  |  |  |  |  | 5 | 7 |  |  |  |  |
| $\bigcirc$ | 42.83 | ${ }^{44.06}$ | 1 | 5 |  | 2 | 2 |  |  |  |  |  |  |  | 4 | 6 |  |  |  |  |  |  |  |
| 2 | 21.08 | 21.88 |  |  |  | $\bigcirc$ |  |  | 31 | 17.69 | 16.22 | 3 | 6 |  |  |  |  | $\stackrel{1}{2}$ | 22.49 17.60 | ${ }^{22} 17.84$ |  |  |  |
| 6 | 21.18 | 20.68 | 2 | 13.27 | 12.48 |  | 37.16 | 36.64 | 5 | 9.42 | ${ }_{8} 8.61$ | 3 | 5 |  | 2 | 34.17 | 35.79 | 4 | 22.31 | 21:97 |  |  |  |
| 8 | 9.52 | 9.52 | 4 | ${ }^{27} 7$ | 26.1? | 4 |  | 45.63 22.09 |  | 15.40 | 6.03 26.04 | $\frac{1}{2}$ | 25.51 | 25.33 | 4 | 18.17 | 16.92 | 6 | 25.32 | 23.36 |  |  |  |
|  |  |  | 6 | 57.16 | 59.89 | ${ }_{6}$ | 21.28 30 | 22.09 29.34 |  | 15.06 | 26.04 | 3 | 20.42 | 19.49 | 6 | 12.80 | 12.35 |  |  |  |  |  |  |
| 0 | 10 |  | 18 | 120.99 | 18.21 |  | 18.26 |  |  |  |  | 7 | 4.62 | 3.84 4.40 | 8 | 17.60 | 10.64 |  |  |  |  |  |  |
|  |  |  | 12 | 8.471 | 10:86 | 12 | 17.50 | 17.89 | 2 | $\checkmark$ |  | 9 | 10.45 | 10.25 |  |  |  |  |  |  |  |  |  |
| 2 | 27.32 | 17.91 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 2 | 20.24 20.00 | 21.18 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 6 | 12.61 | 19.98 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | 16.38 | 15.39 |  |  |  |  |  |  |  |  |  |  |  |  |

Table 3. Interatomic distances and angles with their estimated standard deviations (in parentheses)

$\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2+}$ ions are arranged in alternating layers separated by $c / 4$ [see Fig. 1 $(a)$ ]. Each $\operatorname{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2}$ ion is then surrounded by eight $\mathrm{SO}_{4}^{2-}$ ions lying at the corners of a square prism $\left(5.65 \times 5.65 \times 5.34 \AA^{3}\right)$, and vice versa [Fig. 1(b)]. Every $\mathrm{H}_{2} \mathrm{O}$ molecule of the aquo-cation is hydrogen-bound to two oxygen atoms having equal $z$ coordinates but belonging to different sulphate ions; conversely, each O(sulphate) atom is the acceptor of two hydrogen bonds.

The tetrahedra are contracted parallel to the tetragonal axis, the crystallographic symmetry being $S_{4}$ for the total $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{)^{+}}$group and $D_{2 d}$ for $\mathrm{SO}_{4}^{2-}$. Starting from an idealized structure in which the top and bottom edges of each tetrahedron are perpendicular to $\mathbf{c}$ and lie at $45^{\circ}$ to $\mathbf{a}$ and $\mathbf{b}$, we obtain the real structure by rotating the tetrahedra through $c a .9^{\circ}$ about axes parallel to $\mathbf{c}$. Adjacent tetrahedra of each type are rotated in opposite senses. The bond lengths and angles are listed in Table 3. The standard deviations were computed by assuming r.m.s. isotropic errors for the atomic positions (Cruickshank \& Robertson, 1953). The shortest non-bonded contact between oxygen atoms is $3.47 \AA$.

The ellipsoids representing the thermal parameters (restricted to spheroids in the cases of atoms in special positions) are shown in Fig.2. Numerical details are listed in Table 4. With the usual reservation that the thermal parameters may include contributions from systematic error sources, we conclude that the vibrations of Be are isotropic, that $S$ has a marginally larger amplitude along the $S_{4}$ axis than perpendicular to it, and that the vibrational ellipsoids of both alpha O atoms are prolate spheroids whose major axes are about twice the minor axes. The maximum displacements of the O and $\mathrm{O}_{w}$ atoms occur in directions at $83 \pm 2^{\circ}$ and $78 \pm 2^{\circ}$ to the S-O and Be-O bonds, respectively. We note that the anisotropic thermal parameters provide the only significant differences between the present results and those of a neutron diffraction study of $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ (Chidambaram, 1969).

## Discussion

## Dimensions of the $\mathrm{SO}_{4}^{2-}$ ion

The S-O bond length is $1.465 \pm 0.004 \AA$. Baur (1964c) and Larson (1965) have tabulated twenty crystallo-

(b)

Fig. 1. Stereoscopic diagrams of a portion of the $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ unit-cell $\left(0 \leq z \leq \frac{1}{2}\right)$. (a) $z$ Axis vertical, $x$ axis into paper. (b) $x$ Axis vertical, $z$ axis out of paper.
graphically independent sulphate S-O bonds in five different structures in which S-O bond lengths were reported with standard deviations $<0.01 \AA$. The pattern which emerges from these compilations is that the fewer metal ions and hydrogen-bond donors an O (sulphate) atom has around it, and the more distant such neighbours are, the shorter the S-O bond will be (Larson, 1965). The shortest S-O bond recorded by Baur and by Larson is $1.459 \AA$, the smallest mean of the four in any individual structure is $1.471 \AA$, and the mean of their twenty examples is $1.473 \AA$. According to Baur (1964c), the S-O bond length to be expected when the O(sulphate) atom makes only two hydro-gen-bonded contacts is $1.464 \AA$ - exactly the value found in $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$. These values may be corrected for in-phase differential thermal motion (Busing \& Levy, 1958). The correction involves the assumptions (i) that the sulphate ions behave as rigid bodies, and (ii) that the vibrational parameters represent only vibrational effects. We are therefore reluctant to attach a physical interpretation to the equality of the corrected S-O distance in the present crystal ( $1.482 \AA$ ) and the similarly corrected mean value ( $1.483 \AA$ ) found in Baur's and Larson's surveys.

In $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}$ there are four $\mathrm{O}-\mathrm{S}-\mathrm{O}$ angles of $108 \cdot 6^{\circ}$ and two of $111 \cdot 3^{\circ}$. These values lie in the ranges found in other sulphates (Table 5). While they are consistent with Baur's (1964c) observation that $\mathrm{SO}_{4}^{2-}$ groups in crystalline sulphates are usually distorted from $T_{d}$ symmetry, the data in Table 5 do not support his predicted correlation between the degree of dis-


Fig. 2. Thermal vibration ellipsoids in $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$.

Table 4. Vibrational ellipsoids

|  | Ellipsoid principal axis | r.m.s. displacement (Å) | Inclination to unit-cell axi ( ${ }^{\circ}$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $x$ | ( | $z$ |
| S | A | $0 \cdot 122 \pm 0.003$ | 0 | 90 | 90 |
|  | B | $0 \cdot 122 \pm 0.003$ | 90 | 0 | 90 |
|  | C | $0.153 \pm 0.003$ | 90 | 90 | 0 |
| Be | A | $0 \cdot 141 \pm 0.010$ | 0 | 90 | 90 |
|  | B | $0 \cdot 141 \pm 0 \cdot 010$ | 90 | 0 | 90 |
|  | C | $0 \cdot 126 \pm 0 \cdot 015$ | 90 | 90 | 0 |
| $\mathrm{O}(1)$ | A | $0 \cdot 128 \pm 0 \cdot 007$ | 33 | 82 | 58 |
|  | B | $0 \cdot 136 \pm 0.008$ | 105 | 72 | 81 |
|  | C | $0 \cdot 256 \pm 0.005$ | 119 | 105 | 56 |
| $\mathrm{O}(2)$ | A | $0 \cdot 124 \pm 0.007$ | 87 | 23 | 68 |
|  | B | $0 \cdot 138 \pm 0.007$ | 143 | 101 | 55 |
|  | C | $0.245 \pm 0.005$ | 53 | 110 | 44 |

Table 5. O-S-O angles in sulphate groups (e.s.d.'s in parentheses)

| Crystal | Angular range $\left({ }^{\circ}\right)$ | Bonding of four O(sulphate) atoms | Reference |
| :---: | :---: | :---: | :---: |
| MgSO $4.4 \mathrm{H}_{2} \mathrm{O}$ | 108.2-111.8(0.2) | One accepts 3 H -bonds | Baur, 1962 |
| $\mathrm{FeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ |  | One accepts 2 H -bonds | Baur, 1964a |
|  |  | One accepts 1 H -bond \& coordinates $\mathrm{M}^{2+}$ |  |
|  |  | One coordinates $\mathrm{M}^{2+}$ |  |
| $\mathrm{Li}_{2} \mathrm{SO}_{4} . \mathrm{H}_{2} \mathrm{O}$ | 108.4-110.9 (0.1) | Three have $2 \mathrm{Li}^{+}$neighbours | Larson, 1965 |
|  |  | One has $1 \mathrm{Li}^{+}$neighbour \& accepts 1 H -bond |  |
| $\mathrm{FeSO}_{4} .7 \mathrm{H}_{2} \mathrm{O}$ | 108.4-110.1 (0.3) | Three accept 3 H -bonds | Baur, 1964b |
| $\mathrm{MgSO}_{4} .7 \mathrm{H}_{2} \mathrm{O}$ |  | One accepts 2 H -bonds |  |
|  | 108.6-110.2 (0.3) | Two accept 3 H -bonds | Baur, 1964c |
|  |  | Two accept 2 H -bonds |  |
| $\left[\mathrm{CH}_{3} \mathrm{SC}\left(\mathrm{NH}_{2}\right)_{2}\right]_{2} \mathrm{SO}_{4}$ | 109.0-109.9 (0.4) | Each accepts 2 H -bonds | Stam, 1962 |
| $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}$ | 108.6-111.3 (0.3) | Each accepts 2 H -bonds | Present work |

tortion and the extent to which the environments of $O$ (sulphate) atoms are unequal.

## The $\mathrm{Be}\left(\mathrm{OH}_{2}\right)_{4}^{2+}$ ion

The length of the $\mathrm{H}_{2} \mathrm{O}-\mathrm{Be}$ bond is $1 \cdot 610 \pm 0 \cdot 004 \AA$. Differential vibrational correction, on the assumption that the water molecule is riding upon $\mathrm{Be}^{2+}$, leads to a distance of $1.622 \AA$. This distance agrees with the $\mathrm{Be}-\mathrm{O}$ (acetate) distance found in $\mathrm{Be}_{4} \mathrm{O}\left(\mathrm{CH}_{3} \mathrm{COO}\right)_{6}$, $1 \cdot 624 \AA$ (Tulinsky \& Worthington, 1959). It is significantly shorter than the $\mathrm{Be}-\mathrm{O}$ distances 1.655 and $1.647 \AA$ in BeO (Jeffrey, Parry \& Mazzi, 1956) and the average lengths $1 \cdot 636,1 \cdot 650 \AA$ for $\mathrm{Be}-\mathrm{O}$ and $\mathrm{Be}-(\mathrm{OH})$ bonds reported in a review by Ross (1964).

There is significant angular distortion in the coordination tetrahedron, the two different $\mathrm{O}_{w}-\mathrm{Be}-\mathrm{O}_{w}$ angles being 117.5 and $105 \cdot 6^{\circ}$. There are no strong external bonding forces acting to perturb the shape of the ion from the regular tetrahedron expected for $s p^{3}$ hybridization of the Be orbitals.

## Hydrogen bonds

The direct $\mathrm{O}_{w} \cdots \mathrm{O}$ distances for the two hydrogen bonds formed by each water molecule are 2.697 and $2 \cdot 610 \AA$, at angles of $130^{\circ}$ and $114^{\circ}$ with the $\mathrm{Be}-\mathrm{O}_{w}$ direction, respectively. The angle between the $\mathrm{Be}-\mathrm{O}_{w}$ bond and the $\mathrm{O} \cdots \mathrm{O}_{w} \cdots \mathrm{O}$ plane is $13^{\circ}$. A more detailed description of the hydrogen-bonding cannot be given without more precise values for the H atom positions. We note, however, that all the hydrogen-bonds would be shortened if either the $\mathrm{SO}_{4}$ or $\mathrm{Be}\left(\mathrm{O}_{w}\right)_{4}$ tetrahedra (or both) were given perfect $T_{d}$ symmetry, the S and Be atoms remaining where they are. In the case of the $\mathrm{Be}\left(\mathrm{O}_{w}\right)_{4}$ tetrahedra, this would involve shifting the $\mathrm{O}_{w}$ atoms by $0.113 \AA$ and would make the $\mathrm{Be}-\mathrm{O}_{w}$ and two $\mathrm{O}_{w} \cdots \mathrm{O}$ bonds coplanar. It appears that the actual structure represents a compromise between the tendencies of the Be and O (water) atoms to have tetrahedral bond distributions.

## Additional'observations

We record here two observations which have not yet been fully explored and between which no relationship has been found. They do not appear to have affected the structure analysis.
(1) All of the X-ray powder photographs (see Experimental) showed lines of weaker intensity which could not be attributed to the $\mathrm{BeSO}_{4} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ lattice, and which

Table 6. Unidentified lines on the X-ray powder photographs of $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}(\mathrm{Cu} \mathrm{K} \mathrm{\alpha}$ radiation)

| $\sin \theta$ | $\sin \theta$ | $\sin \theta$ |
| :---: | :---: | :---: |
| $0 \cdot 12773$ | 0.21547 | 0.28164 |
| 0.15008 | 0.21826 | 0.28505 |
| 0.15339 | 0.24387 | 0.32958 |
| 0.18684 | 0.25325 | 0.44690 |
| 0.20310 | 0.26208 |  |

did not correspond with the published patterns of the di-, mono- and anhydrate of $\mathrm{BeSO}_{4}$ (Petersen, Rinn \& Sutton, 1964). The spacings of these extra lines are listed in Table 6.
(2) On the single-crystal photographs, diffuse X-ray scattering was associated with the more intense reflexions. By use of a crystal monochromator this scattering was separated from the effects of impure radiation, and a qualitative determination of the shape of the equal-diffusion surfaces in reciprocal space was made from a series of closely adjacent stationary-crystal photographs. The most intense region, which lay around the (112) reflexion, was examined in greatest detail; here the equal-diffusion surface appeared to be a thin plate, normal to the reciprocal lattice vector and with its maximum possibly slightly displaced from the reciprocal lattice point along that vector.
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# A Neutron Diffraction Determination of the Structure of Beryllium Sulphate Tetrahydrate, $\mathrm{BeSO}_{\mathbf{4}} \cdot \mathbf{4 \mathrm { H } _ { \mathbf { 2 } } \mathrm { O }}$ 

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#### Abstract

The crystal structure of $\mathrm{BeSO}_{4} .4 \mathrm{H}_{2} \mathrm{O}$ (space group $I 4 c 2 ; a=7.990 \AA, c=10.688 \AA ; Z=4$ ) has been directly determined from the intensities of 251 independent three-dimensional single-crystal neutron reflexions with $\sin \theta / \lambda \leq 0.7 \AA^{-1}$. The coordinates of the heavy atoms were obtained by the use of symmetry minimum and minimum functions and the hydrogen atoms were located from a 'weighted heavy atom' three-dimensional neutron scattering density Fourier synthesis. The parameters were further refined by the full-matiix least-squares method to an $R$ value of 0.071 . The positional parameters of the heavy atoms are in excellent agreement with a recent independent X-ray study of the same compound. The S-O distance in the sulphate group is $1.464 \pm 0 \cdot 004 \AA$. Four sulphate oxygens form a tetrahedron around beryllium with a Be-O distarce of $1.618 \pm 0 \cdot 004 \AA$. The structure contains two hydrogen bonds from the water molecule to the sulphate oxygens of length $2 \cdot 684 \pm 0.005 \AA$ and $2 \cdot 617 \pm 0.005 \AA$, the $\mathrm{O}-\mathrm{H}$ distances being $0.971 \pm 0.006 \AA$ and $0.967 \pm 0.006 \AA$ (both uncorrected for thermal motion); the $\mathrm{H}-\mathrm{O}-\mathrm{H}$ angle is $112 \cdot 7 \pm 0.8^{\circ}$.


## Introduction

The crystal structure of beryllium sulphate tetrahydrate was first studied using X-rays by Schonefeld (1931), who described the structure (in the space group $I 4 / \mathrm{mcm}$ ) as consisting of planar $\mathrm{SO}_{4}$ and tetrahedral $\mathrm{Be}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$ groups. Beevers \& Lipson (1932) reinvestigated the structure and showed that the true space group is $I \overline{4} c 2$; the postulated structure had tetrahedral $\mathrm{SO}_{4}$ and $\mathrm{Be}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$ groups linked together by strong hydrogen bonds of length $2.56 \AA$ between the water and sulphate oxygens. That the hydrogen bonds were strong was also suggested from the study by Glemser \& Hartert (1955) of the correlation between the $\mathrm{O}-\mathrm{H}$ stretching frequency and the $\mathrm{O}-\mathrm{-} \mathrm{O}$ distance in hydrates. Our aim in taking up the neutron diffraction study of this crystal was to investigate the effect of strong hydrogen bonds (if these were present in the crystal) on the shape of the water molecule.

Recently, Dance \& Freeman (1969) have refined this structure using X-rays and this has also provided an opportunity to compare the structural parameters determined by the two methods.

## Experimental

The neutron diffraction intensity data were collected at the CIRUS reactor in Trombay using the doublecrystal diffractometer 'DCD' (Chidambaram, Sequeira \& Sikka, 1964) on which a General Electric single crystal orienter had been mounted in the 'symmetrical' position. The crystal used for data collection was parallelepiped in shape, of weight 25 mg , and was mounted with the longest dimension ( 8.4 mm ) corresponding to the [110] direction along the $\varphi$ axis. Before mounting the crystal on the goniometer, it was coated with an adhesive (brand name 'Stickfast') and then repeatedly dipped in liquid nitrogen to reduce extinction affects. The intensities of all the 275 independent reflexions up to $\sin \theta / \lambda=\dot{0} \cdot 70 \AA^{-1}$ were then measured at a wavelength of $1.031 \AA$ using the $\theta-2 \theta$ scan technique. The orientation angles, $2 \theta, \chi, \varphi$, for each reflexion were set by hand. 24 reflexions had intensities less than their standard deviations computed from the counting statistics, and were treated as unobservables.

The integrated intensities were corrected for absorption ( $\mu=1.78 \mathrm{~cm}^{-1}$, as measured experimentally). The


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